Quoniam
QUANTITATIVE INVESTMENT ENGINEERING

ABSTRACT – DISSERTATION

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ON CONTINUOUS TIME TRADING OF A SMALL INVESTOR IN A LIMIT ORDER MARKET

We provide a mathematical framework to model continuous time trading in limit order markets of a small investor whose transactions have no impact on order book dynamics. The investor can continuously place market and limit orders. A market order is executed immediately at the best currently available price, whereas a limit order is stored until it is executed at its limit price or canceled. The limit orders can be chosen from a continuum of limit prices.

In this framework we show how elementary strategies (hold limit orders with only finitely many different limit prices and rebalance at most finitely often) can be extended in a suitable way to general continuous time strategies containing orders with infinitely many different limit prices. The general limit buy order strategies are predictable processes with values in the set of nonincreasing demand functions (not necessarily left- or right-continuous in the price variable). It turns out that this family of strategies is closed and any element can be approximated by a sequence of elementary strategies.

Furthermore, we study Merton's portfolio optimization problem in a specific instance of this framework. Assuming that the risky asset evolves according to a geometric Brownian motion, a proportional bid-ask spread, and Poisson execution times for the limit orders of the small investor, we show that the optimal strategy consists in using market orders to keep the proportion of wealth invested in the risky asset within certain boundaries, similar to the result for proportional transaction costs, while within these boundaries limit orders are used to profit from the bid-ask spread.

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