Response functions, trading strategies, and random matrices: Analysis of large fluctuations and correlations in stock price diffusion

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Abstract – Dissertation

In this thesis, we analyze and explain various properties of stock price changes.

The change of a stock price in a given time interval is composed of many price changes resulting from single trades. Thus, the up and down movements of a stock price can be seen as analogous to the classic diffusion of a particle: if the particle moves due to random collisions with other particles, the displacement after some time is determined by the sum of the displacements between the collisions.

Under certain conditions, the values of such sums are Gaussian distributed. In contrast, extreme price movements such as those of "Black Monday" in 1987, when the S&P500 index fell by about 20% within one day, are so much larger than ordinary price movements that they cannot be accounted for by a Gaussian distribution. One could classify such events as "outliers" that reflect an abnormal market behavior. However, our empirical analysis reveals self-similar features in the time series of price changes, meaning that price changes exhibit similar characteristics on many scales. In particular, a huge price change induces a series of large price changes whose rate decreases over the following months. In a similar way, some of these subsequent large price changes themselves induce further series of intermediate price changes in the following days. Hence, the mechanisms connected to huge price changes seem to be similar for smaller price changes, raising the possibility that these same mechanisms might also underlie ordinary price movements. This picture is supported by the widely accepted finding that the tail of the distribution of stock returns, i.e. changes of the logarithm of the stock price, follows a power law that describes intermediate returns as well as extreme events.

Though extreme returns seem to be "ordinary" in the sense that they are connected to the same mechanisms as smaller returns, it is still an open question how returns can occur that are much larger than can be accounted for by a Gaussian distribution. In fixed time intervals, where the large price movements described above take place, the return is determined by two factors: the number of trades in the respective interval and the magnitude of the returns due to single trades (tick returns). In order to better distinguish between these two effects, we focus on intervals with a fixed number of trades, rather than on intervals defined by their actual length in units of time. Interestingly, also here we find unusually large returns, resulting from the concurrence of two things: (i) in the respective interval, the average tick return is large and (ii) most trades change the price in the same direction. We show that a statistical model incorporating the average tick return and the direction of tick returns can reproduce the distribution of stock returns in the studied intervals.

While this analysis explains in detail how large stock returns are composed, we examine in a further study why these strong returns occur. It is a reasonable assumption that, besides the influence of news, prices change in response to an imbalance between supply and demand. This imbalance can be quantified by volume imbalance, defined as the difference between the volume (number of shares) of buy and sell orders in a given time interval. On a given volume imbalance, the stock price reacts with a price change that is determined by the price impact function. We reconstruct this function in each time interval from data containing information about all orders present in the market. Here, we show that the time-varying slope of the price impact function is responsible for very large returns. Though in each time interval the price moves due to the volume imbalance, extremely large returns occur only when the price impact function is steeper than average.

If prices change in response to trades, there seems to be a paradox: the signs of orders, indicating whether it is a buy or a sell order, are long-term correlated, whereas the returns resulting from the

execution of these orders exhibit only short-term correlations with a characteristic time of a few minutes. In order to understand this paradox, we model trading strategies and show that uncorrelated stock price changes appear naturally as soon as someone uses the correlations in the orders to make profit.

After studying time correlations in the returns, we also investigate a tool that can be used to analyze cross-correlations between finite time series. Since their length is limited, even uncorrelated time series exhibit spurious cross-correlations resulting from random co-movements that do not reflect the real interactions. We show that a hypothesis test based on random matrix theory can distinguish spurious correlations from real correlations, which we demonstrate using numerical simulations.